Chapter 8- Quadrilaterals
Exercise 8.1

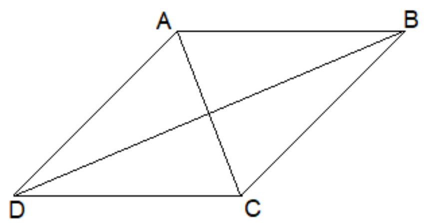
Question 1: The angles of a quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

Answer: Let the angles of the quadrilateral be 3x, 5x, 9x and 13x.
therefore, 3x + 5x + 9x + 13x = 360° [as we know angle sum property of a quadrilateral]
or, 30x = 360°
or, $x = \frac{360°}{30} = 12°$

thus, 3x = 3 x 12° = 36°
5x = 5 x 12° = 60°
9x = 9 x 12° = 108°
13a = 13 x 12° = 156°
Hence, the required angles of the quadrilateral are 36°, 60°, 108° and 156°.

Question 2: If the diagonals of a parallelogram are equal, then show that it is a rectangle.

Answer: Let ABCD is a parallelogram and AC = BD.



In ΔABC and ΔDCB,
AC = DB [Given]
AB = DC [Opposite sides of a parallelogram]
BC = CB [Common]

therefore, ΔABC ≅ ΔDCB [By SSS congruency]
or, ∠ABC = ∠DCB [By C.P.C.T.] ……………………………..(1)

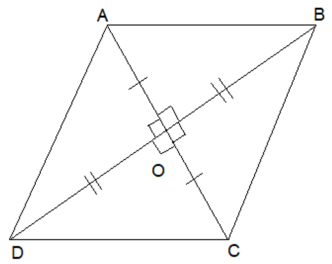
Now, AB || DC and BC is a transversal. [As we know that, ABCD is a parallelogram]
therefore, ∠ABC + ∠DCB = 180° ………………………………………(2) [Co-interior angles]

Now from (1) and (2), we have
∠ABC = ∠DCB = 90°
i.e., ABCD is a parallelogram having an angle equal to 90°.

Hence, ABCD is a rectangle.

Question 3: Show that if the diagonals of a quadrilateral bisect each other at right angles, then it is a rhombus.

Answer: Let ABCD be a quadrilateral such that the diagonals AC and BD bisect each other at O making a right angle.

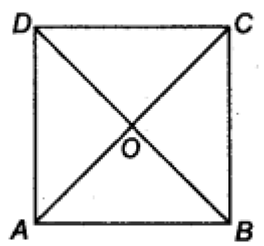


therefore, In ∆AOB and ∆AOD, we have
AO = AO [Common]
OB = OD [O is the mid-point of BD]
∠AOB = ∠AOD [Each 90°]

therefore, ∆AQB ≅ ∆AOD [By,SAS congruency]
hence, AB = AD [By C.P.C.T.] ………………………...(1)
Similarly, AB = BC ………………………………..(2)
BC = CD …………………………………...(3)
CD = DA ……………………………..…(4)
therefore, From (1), (2), (3) and (4), we have
AB = BC = CD = DA
Thus, the quadrilateral ABCD is a rhombus.

Question 4: Show that the diagonals of a square are equal and bisect each other at right angles.

Answer: Let ABCD be a square such that its diagonals AC and BD intersect at O.



i) <u>To prove that the diagonals are equal.</u>
Therefore, we need to prove AC = BD.
In ∆ABC and ∆BAD, we have
AB = BA [Common]
BC = AD [Sides of a square ABCD]

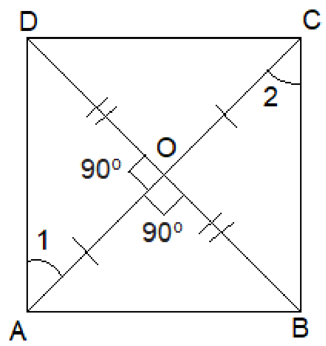
∠ABC = ∠BAD [Each angle is 90°]
hence, ΔABC ≅ ΔBAD [By SAS congruency]
AC = BD [By C.P.C.T.] ………………………………………..……(1)

(ii) To prove diagonals bisect each other.
AD || BC and AC is a transversal. [∵ A square is a parallelogram]
therefore, ∠1 = ∠3
[Alternate interior angles are equal]
Similarly, ∠2 = ∠4
Now, in ΔOAD and ΔOCB, we have
AD = CB [Sides of a square ABCD]
∠1 = ∠3 [Proved]
∠2 = ∠4 [Proved]
therefore, ΔOAD ≅ ΔOCB [By ASA congruency]
⇒ OA = OC and OD = OB [By C.P.C.T.]
i.e., the diagonals AC and BD bisect each other at O…………….(2)

iii) To prove diagonals bisect each other at 90°.
In ΔOBA and ΔODA, we have
OB = OD [Proved]
BA = DA [Sides of a square ABCD]
OA = OA [Common]
therefore, ΔOBA ≅ ΔODA [By SSS congruency]
or, ∠AOB = ∠AOD [By C.P.C.T.] ……………………………………..…(3)
As we know that, ∠AOB and ∠AOD form a linear pair
hence, ∠AOB + ∠AOD = 180°
therefore, ∠AOB = ∠AOD = 90° [By(3)]
or, AC ⊥ BD …………………………………………………………………..(4)
From (1), (2) and (4), we get AC and BD are equal and bisect each other at right angles.

Question 5: Show that if the diagonals of a quadrilateral are equal and bisect each other at right angles, then it is a square.

Answer: Let ABCD be a quadrilateral such that diagonals AC and BD are equal and bisect each other at right angles.



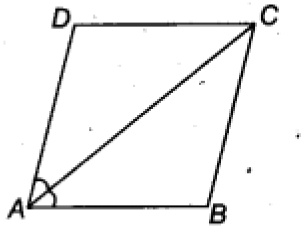
In ∆AOD and ∆AOB, we have
∠AOD = ∠AOB [Each 90°]
AO = AO [Common]
OD = OB [As, O is the midpoint of BD]
therefore, ∆AOD ≅ ∆AOB [By SAS congruency]
or, AD = AB [By C.P.C.……………………(1)
Similarly, we have
AB = BC ……………………………… (2)
BC = CD ……………………………..…(3)
CD = DA …………………………………(4)
From (1), (2), (3) and (4), we have
AB = BC = CD = DA
Therefore, all sides of the Quadrilateral ABCD are equal.

In ∆AOD and ∆COB, we have
AO = CO [Given]
OD = OB [Given]
∠AOD = ∠COB [Vertically opposite angles]
So, ∆AOD ≅ ∆COB [By SAS congruency]
therefore, ∠1 = ∠2 [By C.P.C.T.]
But, they form a pair of alternate interior angles.
hence, AD || BC
Similarly, AB || DC
Therefore, ABCD is a parallelogram.
As we know that, Parallelogram having all its sides equal is a rhombus.
Therefore, ABCD is a rhombus.

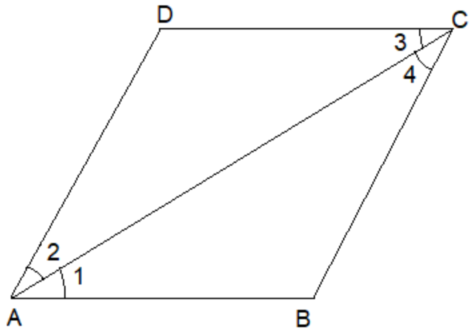
Now, in ∆ABC and ∆BAD, we have
AC = BD [Given]
BC = AD [Proved]
AB = BA [Common]
therefore, ∆ABC ≅ ∆BAD [By SSS congruency]
hence, ∠ABC = ∠BAD [By C.P.C.T.] …………………………(5)
Since AD || BC and AB is a transversal.
thus, ∠ABC + ∠BAD = 180° ………………………………………..(6) [Co – interior angles]
or, ∠ABC = ∠BAD = 90° [By(5) & (6)]
So, rhombus ABCD is having one angle equal to 90°.
Thus, ABCD is a square.

Question 6: Diagonal AC of a parallelogram ABCD bisects ∠A (see figure). Show that (i) it bisects ∠C also,

(ii) ABCD is a rhombus.



Answer:



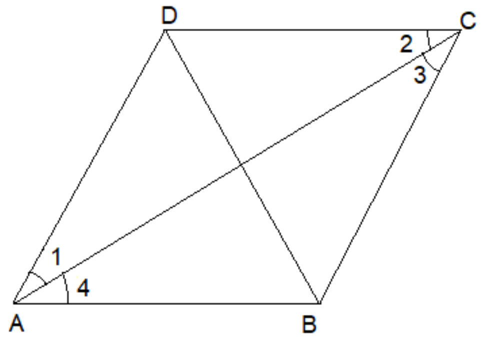
We have a parallelogram ABCD whose diagonal AC bisects ∠A
hence, ∠DAC = ∠BAC

(i) Since, ABCD is a parallelogram.
Therefore, AB || DC and AC is a transversal.
and ∠1 = ∠3 ……………………………………………(1) [Alternate interior angles are equal]
Also, BC || AD and AC is a transversal.
and ∠2 = ∠4 ………………………………………………(2) [Alternate interior angles are equal]
Also, ∠1 = ∠2 ……………………………………….…(3) [since AC bisects ∠A]
From (1), (2) and (3), we have
∠3 = ∠4
Hence, AC bisects ∠C.

(ii) In ΔABC, we have
∠1 = ∠4 [From (2) and (3)]
or, BC = AB ………………………………………………..(4) [Sides opposite to equal angles of a Δ are equal]
Similarly, AD = DC ……………………………………..……..(5)
But, ABCD is a parallelogram. [Given]
therefore, AB = DC ………………………………………….(6)
From (4), (5) and (6), we have
AB = BC = CD = DA
Thus, ABCD is a rhombus.

Question 7: ABCD is a rhombus. Show that diagonal AC bisects ∠Aas well as ∠C and diagonal BD bisects ∠B as well AS ∠D.

Answer:



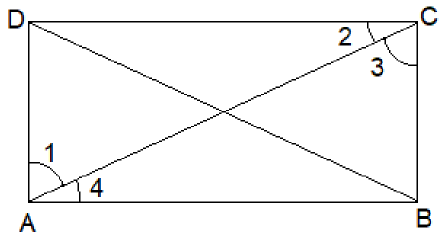
The given ABCD is a rhombus therefore, AB = BC = CD = DA and also, AB || CD and AD || BC

Now, from the diagram CD = AD or, ∠1 = ∠2 ……………………………..….(1) [Angles opposite to equal sides of a triangle are equal]
Also, AD || BC and AC is the transversal. [Every rhombus is a parallelogram]
or, ∠1 = ∠3 ……………………………………………..…(2) [Alternate interior angles are equal]
From (1) and (2), we have
∠2 = ∠3 ………………………………………………..(3)
Since AB || DC and AC is transversal.
therefore, ∠2 = ∠4 ……………………………………(4) [Alternate interior angles are equal]
From (1) and (4), we have ∠1 = ∠4
Therefore, AC bisects ∠C as well as ∠A.

Again, AB = CB or, ∠3 = ∠4 ……………………………(4) [Angles opposite to equal sides of a triangle are equal]
Also, AB || DC and BD is the transversal. [Every rhombus is a parallelogram]
or, ∠2 = ∠4 ……………………………………………(5) [Alternate interior angles are equal]
From (4) and (5), we have,
∠1 = ∠4 ……………………………………………………(6)
Since AD || BC and BD is transversal.
therefore, ∠1 = ∠3 ……………………………………(7) [Alternate interior angles are equal]
From (4) and (7), we have ∠2 = ∠3
Therefore, BD bisects ∠B, as well as ∠D.

Question 8: ABCD is a rectangle in which diagonal AC bisects ∠A as well as ∠C. Show that
(i) ABCD is a square
(ii) diagonal BD bisects ∠B as well as ∠D.

Answer:

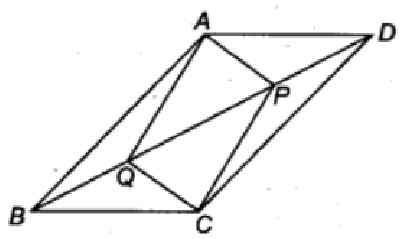


We have a rectangle ABCD such that AC bisects ∠A as well as ∠C.
i.e., ∠1 = ∠4 and ∠2 = ∠3 ……………………………………………..(1)

i) We know that every rectangle is a parallelogram.
Therefore, ABCD is a parallelogram.
Or, AB || CD and AC is a transversal.
therefore, ∠2 = ∠4 ………………………………………(2)[Alternate interior angles are equal]
From (1) and (2), we have
∠3 = ∠4
In ΔABC, ∠3 = ∠4, hence, AB = BC [Sides opposite to equal angles of A are equal]
Similarly, CD = DA
So, ABCD is a rectangle having adjacent sides equal.
Hence, ABCD is a square.

ii) Since ABCD is a square and diagonals of a square bisect the opposite angles.
So, BD bisects ∠B as well as ∠D.

Question 9: In parallelogram ABCD, two points P and Q are taken on diagonal BD such that DP = BQ (see figure). Show that,
i) ΔAPD ≅ ΔCQB
ii) AP = CQ
iii) ΔAQB ≅ ΔCPD
iv) AQ = CP
v) APCQ is a parallelogram



Answer: We have a parallelogram ABCD, BD is the diagonal and points P and Q are such that PD = QB

(i) Since AD || BC and BD is a transversal.
therefore, ∠ADB = ∠CBD [Alternate interior angles are equal]
or, ∠ADP = ∠CBQ
Now, in ΔAPD and ΔCQB, we have
AD = CB [Opposite sides of a parallelogram ABCD are equal]

PD = QB [Given]
∠ADP = ∠CBQ [Proved]
hence, ∆APD ≅ ∆CQB [By SAS congruency]

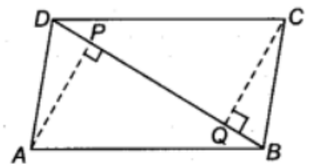
(ii) Since, ∆APD ≅ ∆CQB [Proved]
or, AP = CQ [By C.P.C.T.]

(iii) Since, AB || CD and BD is a transversal therefore, ∠ABD = ∠CDB
or, ∠ABQ = ∠CDP
Now, in ∆AQB and ∆CPD, we have
QB = PD [Given]
∠ABQ = ∠CDP [Proved]
AB = CD [Y Opposite sides of a parallelogram ABCD are equal]
hence, ∆AQB = ∆CPD [By SAS congruency]

(iv) Since, ∆AQB = ∆CPD [Proved]
or, AQ = CP [By C.P.C.T.]

(v) In a quadrilateral ∆PCQ,
Opposite sides are equal. [Proved]
Or, ∆PCQ is a parallelogram.

Question 10: ABCD is a parallelogram, and AP and CQ are perpendiculars from vertices A and C on diagonal BD (see figure). Show that
i) ∆APB ≅ ∆CQD
ii) AP = CQ



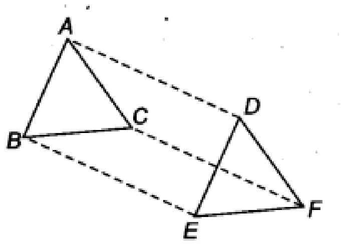
(ii) $AP = CQ$.

Answer: (i) In ∆APB and ∆CQD, we have
∠APB = ∠CQD [Each 90°]
AB = CD [Opposite sides of a parallelogram ABCD are equal]
∠ABP = ∠CDQ [Alternate angles are equal as AB || CD and BD is a transversal]
therefore, ∆APB = ∆CQD [By AAS congruency]

(ii) Since, ∆APB ≅ ∆CQD [Proved in the previous part (i)]
therefore, AP = CQ [By C.P.C.T.]

Question 11: In ∆ABC and ∆DEF, AB = DE, AB || DE, BC – EF and BC || EF. Vertices A, B and C are joined to vertices D, E and F, respectively (see figure).
Show that

(i) quadrilateral ABED is a parallelogram
(ii) quadrilateral BEFC is a parallelogram
(iii) AD || CF and AD = CF
(iv) quadrilateral ACFD is a parallelogram
(v) AC = DF
(vi) ΔABC ≅ ΔDEF



Answer: i) We have AB = DE [Given]
and AB || DE [Given]
i. e., ABED is a quadrilateral in which a pair of opposite sides (AB and DE) are parallel and of equal length.
therefore, ABED is a parallelogram. [proved]

(ii) BC = EF [Given]
and BC || EF [Given]
i.e. BEFC is a quadrilateral in which a pair of opposite sides (BC and EF) are parallel and of equal length.
therefore, BEFC is a parallelogram. [proved]

(iii) ABED is a parallelogram [Proved]
therefore, AD || BE and AD = BE ………………………………(1) [Opposite sides of a parallelogram are equal and parallel] Also, BEFC is a parallelogram. [Proved]

BE || CF and BE = CF …………………….(2)[Opposite sides of a parallelogram are equal and parallel]
From (1) and (2), we have
AD || CF and AD = CF

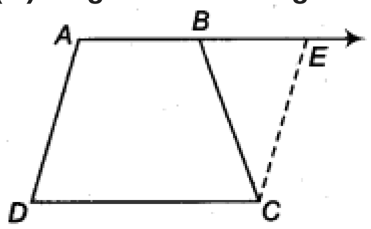
(iv) Since, AD || CF and AD = CF [Proved]
i.e., In quadrilateral ACFD, one pair of opposite sides (AD and CF) are parallel and of equal length.
Therefore, Quadrilateral ACFD is a parallelogram.

(v) Since, ACFD is a parallelogram. [Proved]
So, AC =DF [Opposite sides of a parallelogram are equal]

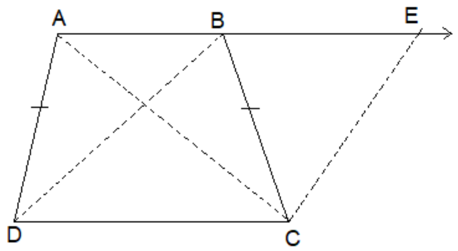
(vi) In ΔABC and ΔDFF, we have
AB = DE [Given]
BC = EF [Given]

AC = DE [Proved in (v) part]
∆ABC ≅ ∆DFF [By SSS congruency]

Question 12: ABCD is a trapezium in which AB || CD and AD = BC (see figure). Show that
(i)∠A=∠B
(ii)∠C=∠D
(iii) ∆ABC ≅ ∆BAD
(iv) diagonal AC = diagonal BD



Answer: We have given a trapezium ABCD in which AB || CD and AD = BC.



(i) Produce AB to E and draw CF || AD as, AB || DC
or, AE || DC Also AD || CF
Or, OECD is a parallelogram.
or, AD = CE …………………………………(1)[Opposite sides of the parallelogram are equal]
But AD = BC ………………………………….…(2) [Given]
By (1) and (2), BC = CF
Now, in ∆BCF, we have BC = CF
⇒ ∠CEB = ∠CBE …………………..(3)[Angles opposite to equal sides of a triangle are equal]
Also, ∠ABC + ∠CBE = 180° …………………………… (4) [Linear pair]
and ∠A + ∠CEB = 180° ……………………..(5) [Co-interior angles of a parallelogram ADCE]
From (4) and (5), we get
∠ABC + ∠CBE = ∠A + ∠CEB
OR, ∠ABC = ∠A [From (3)]
OR, ∠B = ∠A ……………………………………..…(6)

ii) AB || CD and AD is a transversal.
therefore, ∠A + ∠D = 180° ………………………………..(7) [Co-interior angles]
Similarly, ∠B + ∠C = 180° …………………….. (8)
From (7) and (8), we get

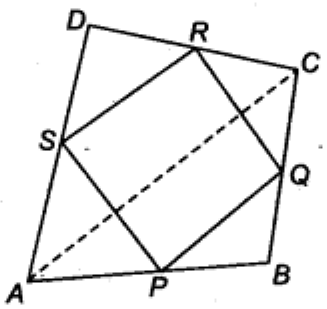
∠A + ∠D = ∠B + ∠C
or, ∠C = ∠D [From (6)]

(iii) In ΔABC and ΔBAD, we have
AB = BA [Common]
BC = AD [Given]
∠ABC = ∠BAD [Proved]
or, ΔABC = ΔBAD [By SAS congruency]

(iv) Since, ΔABC = ΔBAD [Proved] hence, AC = BD [By C.P.C.T.]

Exercise 8.2

Question 1: ABCD is a quadrilateral in which P, Q, R and S are mid-points of the sides AB, BC, CD and DA (see figure). AC is a diagonal. Show that
(i) SR || AC and SR = 12 AC
(ii) PQ = SR
(iii) PQRS is a parallelogram.



Answer: (i) In ΔACD, We have
Therefore, S is the mid-point of AD, and R is the mid-point of CD.
SR = 12AC and SR || AC ……………………………(1)[By mid-point theorem]

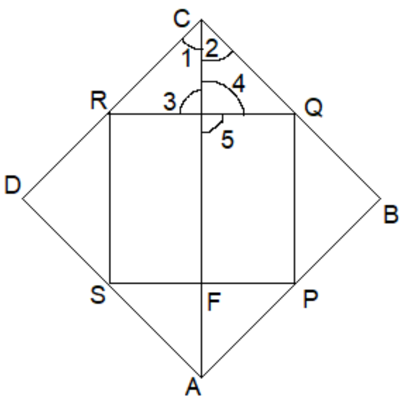
(ii) In ΔABC, P is the mid-point of AB and Q is the mid-point of BC.
PQ = 12AC and PQ || AC ……………………….….(2)[By mid-point theorem]

From (1) and (2), we get
PQ = $\frac{1}{2}$AC = SR and PQ || AC || SR
hence, PQ = SR and PQ || SR

(iii) In a quadrilateral PQRS,
PQ = SR and PQ || SR [Proved]
Therefore, PQRS is a parallelogram.

Question 2: ABCD is a rhombus, and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA, respectively. Show that the quadrilateral PQRS is a rectangle.

Answer:



We have a rhombus ABCD and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. For the convenience of the answer, Join AC.

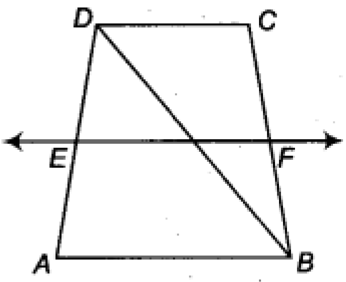
In ∆ABC, P and Q are the mid-points of AB and BC respectively.
therefore, PQ = $\frac{1}{2}$AC and PQ || AC ……………………..…(1) [By mid-point theorem]
In ∆ADC, R and S are the mid-points of CD and DA respectively.
therefore, SR = $\frac{1}{2}$AC and SR || AC ………………………(2) [By mid-point theorem]
From (1) and (2), we get
PQ = $\frac{1}{2}$AC = SR and PQ || AC || SR
or, PQ = SR and PQ || SR
hence, PQRS is a parallelogram. ……………………….….(3)
Now, in ∆ERC and ∆EQC,
∠1 = ∠2 [The diagonals of a rhombus bisect the opposite angles]
CR = CQ [$\frac{CD}{2} = \frac{BC}{2}$]
CE = CE [Common]
Therefore, ∆ERC ≅ ∆EQC [By SAS congruency]
or, ∠3 = ∠4 ……………………………………………………. (4) [By C.P.C.T.]
But ∠3 + ∠4 = 180° ……(5) [Linear pair]
From (4) and (5), we get ∠3 = ∠4 = 90°
Now, ∠QRP = 180° – ∠b [Y Co-interior angles for PQ || AC and EQ is transversal]
But ∠5 = ∠3 [Vertically opposite angles are equal]
thus, ∠5 = 90°
So, ∠RQP = 180° – ∠5 = 90°
So, One angle of parallelogram PQRS is 90°.
Thus, PQRS is a rectangle.

Question 3: ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA, respectively. Show that the quadrilateral PQRS is a rhombus.

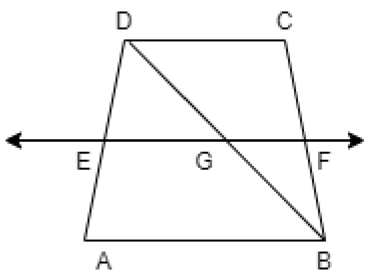
Answer: We have,
Now, on ΔABC, we have
PQ = $\frac{1}{2}$AC and PQ || AC ……………………..…(1) [By mid-point theorem]
Similarly, in ΔADC, we have
SR = $\frac{1}{2}$AC and SR || AC ………………………..(2)

From (1) and (2), we get
PQ = SR and PQ || SR
Therefore, PQRS is a parallelogram.
Now, in ΔPAS and ΔPBQ, we have
$\angle$A = $\angle$B [Each 90°]
AP = BP [P is the mid-point of AB]
AS = BQ [$\frac{1}{2}$AD = $\frac{1}{2}$BC]
therefore, ΔPAS $\cong$ ΔPBQ [By SAS congruency]
hence, PS = PQ [By C.P.C.T.]
Also, PS = QR and PQ = SR [opposite sides of a parallelogram are equal]
So, PQ = QR = RS = SP, i.e., PQRS is a parallelogram having all of its sides equal.
Hence, PQRS is a rhombus.

Question 4: ABCD is a trapezium in which AB || DC, BD is a diagonal and E is the mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F (see figure). Show that F is the mid-point of BC.



Answer:

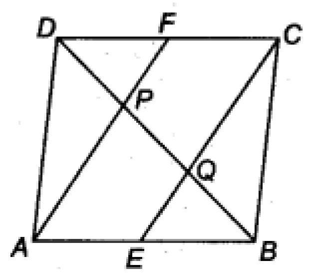


In ΔDAB, we know that E is the mid-point of AD
EG || AB [EF || AB]

Using the converse of mid-point theorem, we get, G is the mid-point of BD.
Again in ABDC,

we have G as the midpoint of BD and GF || DC [AB || DC; EF || AB and GF is a part of EF]
Using the converse of the mid-point theorem, we get, F is the mid-point of BC.

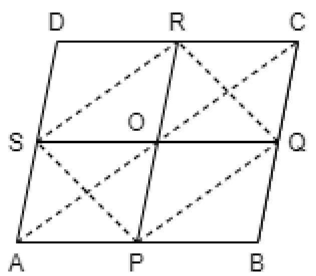
Question 5: In a parallelogram ABCD, E and F are the mid-points of sides AB and CD respectively (see figure). Show that the line segments AF and EC trisect the diagonal BD.



Answer: Since the opposite sides of a parallelogram are parallel and equal.
thus, AB || DC
or, AE || FC …………………………………..…(1)
and AB = DC
Or, $\frac{1}{2}$AB = $\frac{1}{2}$DC
Or, AE = FC ………………………………..(2)
From (1) and (2), we have
AE || PC and AE = PC
Therefore, ΔECF is a parallelogram.
Now, in ΔDQC, we have F is the mid-point of DC and FP || CQ [AF || CE]
thus, DP = PQ ……………………………..…(3)
[By converse of mid-point theorem]
Similarly, in A BAP, E is the mid-point of AB and EQ || AP [AF || CE]
thus, BQ = PQ ………………………………….(4) [By converse of mid-point theorem]
therefore, From (3) and (4), we have
DP = PQ = BQ
So, the line segments AF and EC trisect the diagonal BD.

Question 6: Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Answer: Let ABCD be a quadrilateral, where P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively.
Join PQ, QR, RS and SP.
Let us also join PR, SQ and AC.

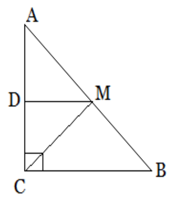


In ΔABC, we have P and Q are the mid-points of AB and BC, respectively.
thus, PQ || AC and PQ = $\frac{1}{2}$AC ……………………..…(1) [By mid-point theorem]
Similarly, RS || AC and RS = $\frac{1}{2}$AC ……………………(2)

thus, By (1) and (2), we get
PQ || RS, PQ = RS
Therefore, PQRS is a parallelogram, and as per rules, the diagonals of a parallelogram bisect each other, i.e., PR and SQ bisect each other. Hence, the line segments joining the midpoints of opposite sides of a quadrilateral ABCD bisect each other.

Question 7: ABC is a triangle right angled at C. A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D. Show that
(i) D is the mid-point of AC
(ii) MD ⊥ AC
(iii) CM = MA = $\frac{1}{2}$AB

Answer:



(i) In ΔACB, we have
MD || BC [Given]
M is the mid-point of AB. [Given]
Now, Using the converse of mid-point theorem,
D is the mid-point of AC.

(ii) Since MD || BC and AC is a transversal.
∠BCA = 90° [Given]
∠MDA = ∠BCA [As Corresponding angles are equal]
∠MDA = 90°
Hence, MD ⊥, AC.

(iii) In ΔADM and ΔCDM, we have
MD = MD [Common]
∠ADM = ∠CDM [Each equal to 90°]
AD = CD [D is the mid-point of AC]
therefore, ΔADM ≅ ΔCDM [By SAS congruency]

thus, MA = MC [By C.P.C.T.] ……………………….(1)
since M is the mid-point of AB [Given]
MA = $\frac{1}{2}$AB ………………………………………..…(2)
From (1) and (2), we have
CM = MA = AB